

Soluções e respostas parciais dos problemas fundamentais

Capítulo 2

$$\begin{aligned}\text{F2.1. } F_R &= \sqrt{(2 \text{ kN})^2 + (6 \text{ kN})^2 - 2(2 \text{ kN})(6 \text{ kN})\cos 105^\circ} \\ &= 6,798 \text{ kN} = 6,80 \text{ kN} \\ \frac{\sin \phi}{6 \text{ kN}} &= \frac{\sin 105^\circ}{6,798 \text{ kN}}, \quad \phi = 58,49^\circ \\ \theta &= 45^\circ + \phi = 45^\circ + 58,49^\circ = 103^\circ\end{aligned}$$

$$\begin{aligned}\text{F2.2. } F_R &= \sqrt{200^2 + 500^2 - 2(200)(500)\cos 140^\circ} \\ &= 666 \text{ N}\end{aligned}$$

$$\begin{aligned}\text{F2.3. } F_R &= \sqrt{600^2 + 800^2 - 2(600)(800)\cos 60^\circ} \\ &= 721,11 \text{ N} = 721 \text{ N} \\ \frac{\sin \alpha}{800} &= \frac{\sin 60^\circ}{721,11}, \quad \alpha = 73,90^\circ \\ \phi &= \alpha - 30^\circ = 73,90^\circ - 30^\circ = 43,9^\circ\end{aligned}$$

$$\begin{aligned}\text{F2.4. } \frac{F_u}{\sin 45^\circ} &= \frac{300}{\sin 105^\circ}; \quad F_u = 219,6 \text{ N} \\ \frac{F_v}{\sin 30^\circ} &= \frac{300}{\sin 105^\circ}; \quad F_v = 155,3 \text{ N}\end{aligned}$$

$$\begin{aligned}\text{F2.5. } \frac{F_{AB}}{\sin 105^\circ} &= \frac{900}{\sin 30^\circ} \\ F_{AB} &= 1738,7 \text{ N}\end{aligned}$$

$$\begin{aligned}\frac{F_{AC}}{\sin 45^\circ} &= \frac{900}{\sin 30^\circ} \\ F_{AC} &= 1272,8 \text{ N}\end{aligned}$$

$$\text{F2.6. } \frac{F}{\sin 30^\circ} = \frac{6}{\sin 105^\circ} \quad F = 3,11 \text{ kN}$$

$$\frac{F_v}{\sin 45^\circ} = \frac{6}{\sin 105^\circ} \quad F_v = 4,39 \text{ kN}$$

$$\begin{aligned}\text{F2.7. } (F_1)_x &= 0 \quad (F_1)_y = 300 \text{ N} \\ (F_2)_x &= -(450 \text{ N})\cos 45^\circ = -318 \text{ N} \\ (F_2)_y &= (450 \text{ N})\sin 45^\circ = 318 \text{ N} \\ (F_3)_x &= \left(\frac{3}{5}\right)600 \text{ N} = 360 \text{ N} \\ (F_3)_y &= \left(\frac{4}{5}\right)600 \text{ N} = 480 \text{ N}\end{aligned}$$

$$\text{F2.8. } F_{Rx} = 300 + 400 \cos 30^\circ - 250\left(\frac{4}{5}\right) = 446,4 \text{ N}$$

$$F_{Ry} = 400 \sin 30^\circ + 250\left(\frac{3}{5}\right) = 350 \text{ N}$$

$$F_R = \sqrt{(446,4)^2 + 350^2} = 567 \text{ N}$$

$$\theta = \tan^{-1} \frac{350}{446,4} = 38,1^\circ \swarrow$$

$$\begin{aligned}\text{F2.9. } \pm (F_R)_x &= \Sigma F_x; \\ (F_R)_x &= -(3,5 \text{ kN}) \cos 30^\circ + 0 + \left(\frac{3}{5}\right)(3 \text{ kN}) \\ &= -1,231 \text{ kN} \\ +\uparrow (F_R)_y &= \Sigma F_y; \\ (F_R)_y &= -(3,5 \text{ kN}) \sin 30^\circ - 2 \text{ kN} - \left(\frac{4}{5}\right)(3 \text{ kN}) \\ &= -6,15 \text{ kN} \\ F_R &= \sqrt{(1,231 \text{ kN})^2 + (6,15 \text{ kN})^2} = 6,272 \text{ kN} \\ \phi &= \tan^{-1} \left(\frac{6,15}{1,231}\right) = 78,68^\circ \\ \theta &= 180^\circ + \phi = 180^\circ + 78,68^\circ = 259^\circ\end{aligned}$$

$$\begin{aligned}\text{F2.10. } \pm (F_R)_x &= \Sigma F_x; \\ 750 \text{ N} &= F \cos \theta + \left(\frac{5}{13}\right)(325 \text{ N}) + (600 \text{ N})\cos 45^\circ \\ +\uparrow (F_R)_y &= \Sigma F_y; \\ 0 &= F \sin \theta + \left(\frac{12}{13}\right)(325 \text{ N}) - (600 \text{ N}) \sin 45^\circ \\ \tan \theta &= 0,6190 \quad \theta = 31,76^\circ = 31,8^\circ \swarrow \\ F &= 236 \text{ N}\end{aligned}$$

$$\begin{aligned}\text{F2.11. } \pm (F_R)_x &= \Sigma F_x; \\ (400 \text{ N})\cos 45^\circ &= F \cos \theta + 250 \text{ N} - \left(\frac{3}{5}\right)450 \text{ N} \\ +\uparrow (F_R)_y &= \Sigma F_y; \\ -(400 \text{ N})\sin 45^\circ &= F \sin \theta - \left(\frac{4}{5}\right)(450 \text{ N}) \\ \tan \theta &= 0,2547 \quad \theta = 14,29^\circ = 14,3^\circ \swarrow \\ F &= 312,5 \text{ N}\end{aligned}$$

$$\begin{aligned}\text{F2.12. } (F_R)_x &= 15\left(\frac{4}{5}\right) + 0 + 15\left(\frac{4}{5}\right) = 24 \text{ kN} \rightarrow \\ (F_R)_y &= 15\left(\frac{3}{5}\right) + 20 - 15\left(\frac{3}{5}\right) = 20 \text{ kN} \uparrow \\ F_R &= 31,2 \text{ kN} \\ \theta &= 39,8^\circ\end{aligned}$$

F2.13. $F_x = 75 \cos 30^\circ \sin 45^\circ = 45,93 \text{ kN}$

$F_y = 75 \cos 30^\circ \cos 45^\circ = 45,93 \text{ kN}$

$F_z = -75 \sin 30^\circ = -37,5 \text{ kN}$

$\alpha = \cos^{-1} \left(\frac{45,93}{75} \right) = 52,2^\circ$

$\beta = \cos^{-1} \left(\frac{45,93}{75} \right) = 52,2^\circ$

$\gamma = \cos^{-1} \left(\frac{-37,5}{75} \right) = 120^\circ$

F2.14. $\cos \beta = \sqrt{1 - \cos^2 120^\circ - \cos^2 60^\circ} = \pm 0,7071$

Requer $\beta = 135^\circ$.

$\mathbf{F} = F\mathbf{u}_F = (500 \text{ N})(-0,5\mathbf{i} - 0,7071\mathbf{j} + 0,5\mathbf{k})$

$= \{-250\mathbf{i} - 354\mathbf{j} + 250\mathbf{k}\} \text{ N}$

F2.15. $\cos^2 \alpha + \cos^2 \times 135^\circ + \cos^2 120^\circ = 1$

$\alpha = 60^\circ$

$\mathbf{F} = F\mathbf{u}_F = (500 \text{ N})(-0,5\mathbf{i} - 0,7071\mathbf{j} + 0,5\mathbf{k})$

$= \{250\mathbf{i} - 354\mathbf{j} - 250\mathbf{k}\} \text{ N}$

F2.16. $F_z = (250 \text{ N}) \sin 45^\circ = 176,78 \text{ N}$

$F' = (250 \text{ N}) \cos 45^\circ = 176,78 \text{ N}$

$F_x = \left(\frac{3}{5}\right)(176,78 \text{ N}) = 106,1 \text{ N}$

$F_y = \left(\frac{4}{5}\right)(176,78 \text{ N}) = 141,4 \text{ N}$

$\mathbf{F} = \{-106,1\mathbf{i} + 141,4\mathbf{j} + 176 + 8\mathbf{k}\} \text{ N}$

F2.17. $F_z = (750 \text{ N}) \sin 45^\circ = 530,33 \text{ N}$

$F' = (750 \text{ N}) \cos 45^\circ = 530,33 \text{ N}$

$F_x = (530,33 \text{ N}) \cos 60^\circ = 265,1 \text{ N}$

$F_y = (530,33 \text{ N}) \sin 60^\circ \text{ N} = 459,3 \text{ N}$

$\mathbf{F}_2 = \{265\mathbf{i} - 459\mathbf{j} + 530\mathbf{k}\} \text{ N}$

F2.18. $\mathbf{F}_1 = \left(\frac{4}{5}\right)(2,5 \text{ kN})\mathbf{j} + \left(\frac{3}{5}\right)(2,5 \text{ kN})\mathbf{k}$

$= \{2\mathbf{j} + 1,5\mathbf{k}\} \text{ kN}$

$\mathbf{F}_2 = [(4 \text{ kN}) \cos 45^\circ] \cos 30^\circ \mathbf{i}$

$+ [(4 \text{ kN}) \cos 45^\circ] \sin 30^\circ \mathbf{j}$

$+ (4 \text{ kN}) \sin 45^\circ (-\mathbf{k})$

$= (2,45\mathbf{i} + 1,41\mathbf{j} - 2,83\mathbf{k}) \text{ kN}$

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = \{2,45\mathbf{i} + 3,41\mathbf{j} - 1,33\mathbf{k}\} \text{ kN}$

F2.19. $\mathbf{r}_{AB} = \{-6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}\} \text{ m}$

$r_{AB} = \sqrt{(-6 \text{ m})^2 + (6 \text{ m})^2 + (3 \text{ m})^2} = 9 \text{ m}$

$\alpha = 132^\circ, \beta = 48,2^\circ, \gamma = 70,5^\circ$

F2.20. $\mathbf{r}_{AB} = \{-2\mathbf{i} + 1\mathbf{j} + 2\mathbf{k}\} \text{ m}$

$r_{AB} = \sqrt{(-2)^2 + (1)^2 + (2)^2} = 3 \text{ m}$

$\alpha = \cos^{-1} \left(\frac{-2 \text{ m}}{3 \text{ m}} \right) = 131,8^\circ$

$\theta = 180^\circ - 131,8^\circ = 48,2^\circ$

F2.21. $\mathbf{r}_B = \{2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}\} \text{ m}$

$\mathbf{F}_B = F_B \mathbf{u}_B$

$= (630 \text{ N}) \left(\frac{2}{7}\mathbf{i} + \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right)$

$= \{180\mathbf{i} + 270\mathbf{j} - 540\mathbf{k}\} \text{ N}$

F2.22. $\mathbf{F} = F\mathbf{u}_{AB} = 900 \text{ N} \left(-\frac{4}{9}\mathbf{i} + \frac{7}{9}\mathbf{j} - \frac{4}{9}\mathbf{k} \right)$

$= \{-400\mathbf{i} + 700\mathbf{j} - 400\mathbf{k}\} \text{ N}$

F2.23. $\mathbf{F}_B = F_B \mathbf{u}_B$

$= (840 \text{ N}) \left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right)$

$= \{360\mathbf{i} - 240\mathbf{j} - 720\mathbf{k}\} \text{ N}$

$\mathbf{F}_C = F_C \mathbf{u}_C$

$= (420 \text{ N}) \left(\frac{2}{7}\mathbf{i} + \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right)$

$= \{120\mathbf{i} + 180\mathbf{j} - 360\mathbf{k}\} \text{ N}$

$F_R = \sqrt{(480 \text{ N})^2 + (-60 \text{ N})^2 + (-1080 \text{ N})^2}$

$= 1,18 \text{ kN}$

F2.24. $\mathbf{F}_B = F_B \mathbf{u}_B$

$= (3 \text{ kN}) \left(-\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k} \right)$

$= \{-1\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}\} \text{ kN}$

$\mathbf{F}_C = F_C \mathbf{u}_C$

$= (2,45 \text{ kN}) \left(-\frac{6}{7}\mathbf{i} + \frac{3}{7}\mathbf{j} - \frac{2}{7}\mathbf{k} \right)$

$= \{-2,1\mathbf{i} + 1,05\mathbf{j} - 0,7\mathbf{k}\} \text{ kN}$

$\mathbf{F}_R = +\mathbf{F}_B + \mathbf{F}_C = \{-3,1\mathbf{i} + 3,05\mathbf{j} - 2,7\mathbf{k}\} \text{ kN}$

F2.25. $\mathbf{u}_{AO} = -\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$

$\mathbf{u}_F = -0,5345\mathbf{i} + 0,8018\mathbf{j} + 0,2673\mathbf{k}$

$\theta = \cos^{-1} (\mathbf{u}_{AO} \cdot \mathbf{u}_F) = 57,7^\circ$

F2.26. $\mathbf{u}_{AB} = -\frac{3}{5}\mathbf{j} + \frac{4}{5}\mathbf{k}$

$\mathbf{u}_F = \frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$

$\theta = \cos^{-1} (\mathbf{u}_{AB} \cdot \mathbf{u}_F) = 68,9^\circ$

F2.27. $\mathbf{u}_{OA} = \frac{12}{13}\mathbf{i} + \frac{5}{13}\mathbf{j}$

$\mathbf{u}_{OA} \cdot \mathbf{j} = u_{OA}(1) \cos \theta$

$\cos \theta = \frac{5}{13}; \theta = 67,4^\circ$

F2.28. $\mathbf{u}_{OA} = \frac{12}{13}\mathbf{i} + \frac{5}{13}\mathbf{j}$

$\mathbf{F} = F\mathbf{u}_F = [650\mathbf{j}] \text{ N}$

$F_{OA} = \mathbf{F} \cdot \mathbf{u}_{OA} = 250 \text{ N}$

$\mathbf{F}_{OA} = F_{OA} \mathbf{u}_{OA} = \{231\mathbf{i} + 96,2\mathbf{j}\} \text{ N}$

F2.29.

$$\begin{aligned} \mathbf{F} &= (400 \text{ N}) \frac{\{4\mathbf{i} + 1\mathbf{j} - 6\mathbf{k}\} \text{ m}}{\sqrt{(4 \text{ m})^2 + (1 \text{ m})^2 + (-6 \text{ m})^2}} \\ &= \{219,78\mathbf{i} + 54,94\mathbf{j} - 329,67\mathbf{k}\} \text{ N} \\ \mathbf{u}_{AO} &= \frac{\{-4\mathbf{j} - 6\mathbf{k}\} \text{ m}}{\sqrt{(-4 \text{ m})^2 + (-6 \text{ m})^2}} \\ &= -0,5547\mathbf{j} - 0,8321\mathbf{k} \\ (F_{AO})_{\text{proj}} &= \mathbf{F} \cdot \mathbf{u}_{AO} = 244 \text{ N} \end{aligned}$$

F2.30.

$$\begin{aligned} \mathbf{F} &= [(-3 \text{ kN}) \cos 60^\circ] \sin 30^\circ \mathbf{i} \\ &\quad + [(3 \text{ kN}) \cos 60^\circ] \cos 30^\circ \mathbf{j} \\ &\quad + [(3 \text{ kN}) \sin 60^\circ] \mathbf{k} \\ &= \{-0,75\mathbf{i} + 1,299\mathbf{j} + 2,598\mathbf{k}\} \text{ kN} \\ \mathbf{u}_A &= -\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k} \\ (F_A)_{\text{proj}} &= \mathbf{F} \cdot \mathbf{u}_A = 2,232 \text{ kN} \\ (F_A)_{\text{per}} &= \sqrt{(3 \text{ kN})^2 - (2,232 \text{ kN})^2} \\ &= 2,00 \text{ kN} \end{aligned}$$

Capítulo 3

F3.1. $\pm \Sigma F_x = 0; \frac{4}{5} F_{AC} - F_{AB} \cos 30^\circ = 0$

$+ \uparrow \Sigma F_y = 0; \frac{3}{5} F_{AC} + F_{AB} \sin 30^\circ - 2,75 \text{ kN} = 0$

$F_{AB} = 2,39 \text{ kN}$

$F_{AC} = 2,59 \text{ kN}$

F3.2. $+ \uparrow \Sigma F_y = 0; -2(7,5) \sin \theta + 3,5 = 0$

$\theta = 13,5^\circ$

$L_{ABC} = 2 \left(\frac{1,5 \text{ m}}{\cos 13,5^\circ} \right) = 3,09 \text{ m}$

F3.3. $\pm \Sigma F_x = 0; T \cos \theta - T \cos \phi = 0$

$\phi = 0$

$+ \uparrow \Sigma F_y = 0; 2T \sin \theta - 49,05 \text{ N} = 0$

$\theta = \tan^{-1} \left(\frac{0,15 \text{ m}}{0,2 \text{ m}} \right) = 36,87^\circ$

$T = 40,9 \text{ N}$

F3.4. $+ \nearrow \Sigma F_x = 0; \frac{4}{5} (F_{sp}) - 5(9,81) \sin 45^\circ = 0$

$F_{sp} = 43,35 \text{ N}$

$F_{sp} = k(l - l_0); 43,35 = 200(0,5 - l_0)$

$l_0 = 0,283 \text{ m}$

F3.5. $+ \uparrow \Sigma F_y = 0; (392,4 \text{ N}) \sin 30^\circ - m_A(9,81) = 0$

$m_A = 20 \text{ kg}$

F3.6. $+ \uparrow \Sigma F_y = 0; T_{AB} \sin 15^\circ - 10(9,81) \text{ N} = 0$

$T_{AB} = 379,03 \text{ N} = 379 \text{ N}$

$\pm \Sigma F_x = 0; T_{BC} - 379,03 \text{ N} \cos 15^\circ = 0$

$T_{BC} = 366,11 \text{ N} = 366 \text{ N}$

$\pm \Sigma F_x = 0; T_{CD} \cos \theta - 366,11 \text{ N} = 0$

$+ \uparrow \Sigma F_y = 0; T_{CD} \sin \theta - 15(9,81) \text{ N} = 0$

$T_{CD} = 395 \text{ N}$

$\theta = 21,9^\circ$

F3.7. $\Sigma F_x = 0; \left[\left(\frac{3}{5} \right) F_3 \right] \left(\frac{3}{5} \right) + 600 \text{ N} - F_2 = 0$

$\Sigma F_y = 0; \left(\frac{4}{5} \right) F_1 - \left[\left(\frac{3}{5} \right) F_3 \right] \left(\frac{4}{5} \right) = 0$

$\Sigma F_z = 0; \left(\frac{4}{5} \right) F_3 + \left(\frac{3}{5} \right) F_1 - 900 \text{ N} = 0$

$F_3 = 776 \text{ N}$

$F_1 = 466 \text{ N}$

$F_2 = 879 \text{ N}$

F3.8. $\Sigma F_z = 0; F_{AD} \left(\frac{4}{5} \right) - 900 = 0$

$F_{AD} = 1125 \text{ N} = 1,125 \text{ kN}$

$\Sigma F_y = 0; F_{AC} \left(\frac{4}{5} \right) - 1125 \left(\frac{3}{5} \right) = 0$

$F_{AC} = 843,75 \text{ N} = 844 \text{ N}$

$\Sigma F_x = 0; F_{AB} - 843,75 \left(\frac{3}{5} \right) = 0$

$F_{AB} = 506,25 \text{ N} = 506 \text{ N}$

F3.9. $\mathbf{F}_{AD} = F_{AD} \left(\frac{\mathbf{r}_{AD}}{r_{AD}} \right) = \frac{1}{3} F_{AD} \mathbf{i} - \frac{2}{3} F_{AD} \mathbf{j} + \frac{2}{3} F_{AD} \mathbf{k}$

$\Sigma F_z = 0; \frac{2}{3} F_{AD} - 600 = 0$

$F_{AD} = 900 \text{ N}$

$\Sigma F_y = 0; F_{AB} \cos 30^\circ - \frac{2}{3}(900) = 0$

$F_{AB} = 692,82 \text{ N} = 693 \text{ N}$

$\Sigma F_x = 0; \frac{1}{3}(900) + 692,82 \sin 30^\circ - F_{AC} = 0$

$F_{AC} = 646,41 \text{ N} = 646 \text{ N}$

F3.10. $\mathbf{F}_{AC} = F_{AC} \{-\cos 60^\circ \sin 30^\circ \mathbf{i}$

$+ \cos 60^\circ \cos 30^\circ \mathbf{j} + \sin 60^\circ \mathbf{k}\}$

$= -0,25 F_{AC} \mathbf{i} + 0,4330 F_{AC} \mathbf{j} + 0,8660 F_{AC} \mathbf{k}$

$\mathbf{F}_{AD} = F_{AD} \{\cos 120^\circ \mathbf{i} + \cos 120^\circ \mathbf{j} + \cos 45^\circ \mathbf{k}\}$

$= -0,5 F_{AD} \mathbf{i} - 0,5 F_{AD} \mathbf{j} + 0,7071 F_{AD} \mathbf{k}$

$\Sigma F_y = 0; 0,4330 F_{AC} - 0,5 F_{AD} = 0$

$\Sigma F_z = 0; 0,8660 F_{AC} + 0,7071 F_{AD} - 300 = 0$

$F_{AD} = 175,74 \text{ N} = 176 \text{ N}$

$F_{AC} = 202,92 \text{ N} = 203 \text{ N}$

$\Sigma F_x = 0; F_{AB} - 0,25(202,92) - 0,5(175,74) = 0$

$F_{AB} = 138,60 \text{ N} = 139 \text{ N}$

$$\begin{aligned}
 \text{F3.11. } \mathbf{F}_B &= F_B \left(\frac{\mathbf{r}_{AB}}{r_{AB}} \right) \\
 &= F_B \left[\frac{\{-1,8\mathbf{i} + 0,9\mathbf{j} + 0,6\mathbf{k}\} \text{ m}}{\sqrt{(-1,8 \text{ m})^2 + (0,9 \text{ m})^2 + (0,6 \text{ m})^2}} \right] \\
 &= -\frac{6}{7}F_B\mathbf{i} + \frac{3}{7}F_B\mathbf{j} + \frac{2}{7}F_B\mathbf{k} \\
 \mathbf{F}_C &= F_C \left(\frac{\mathbf{r}_{AC}}{r_{AC}} \right) \\
 &= F_C \left[\frac{\{-1,8\mathbf{i} - 0,6\mathbf{j} + 0,9\mathbf{k}\} \text{ m}}{\sqrt{(-1,8 \text{ m})^2 + (-0,6 \text{ m})^2 + (0,9 \text{ m})^2}} \right] \\
 &= -\frac{6}{7}F_C\mathbf{i} - \frac{2}{7}F_C\mathbf{j} + \frac{3}{7}F_C\mathbf{k} \\
 \mathbf{F}_D &= F_D\mathbf{i} \\
 \mathbf{W} &= \{-75(9,81)\mathbf{k}\} \text{ N} \\
 \Sigma F_x &= 0; -\frac{6}{7}F_B - \frac{6}{7}F_C + F_D = 0 \\
 \Sigma F_y &= 0; \frac{3}{7}F_B - \frac{2}{7}F_C = 0 \\
 \Sigma F_z &= 0; \frac{2}{7}F_B + \frac{3}{7}F_C - 75 \times 9,81 = 0 \\
 F_B &= 729,3 \text{ N} \\
 F_C &= 1,5(729,3 \text{ N}) = 1188,5 \text{ N} \\
 F_D &= 1697,8 \text{ N}
 \end{aligned}$$

Capítulo 4

$$\begin{aligned}
 \text{F4.1. } \zeta + M_O &= 3 \sin 50^\circ(1,5) + 3 \cos 50^\circ(1,5) \\
 &= 3,74 \text{ kN} \\
 \text{F4.2. } \zeta + M_O &= -\left(\frac{4}{5}\right)(100 \text{ N})(2 \text{ m}) - \left(\frac{3}{5}\right)(100 \text{ N})(5 \text{ m}) \\
 &= -460 \text{ N} \cdot \text{m} = 460 \text{ N} \cdot \text{m} \curvearrowright \\
 \text{F4.3. } \zeta + M_O &= [(300 \text{ N}) \sin 30^\circ][(0,4 \text{ m} + (0,3 \text{ m}) \cos 45^\circ] \\
 &\quad - [(300 \text{ N}) \cos 30^\circ][(0,3 \text{ m}) \sin 45^\circ] \\
 &= 36,7 \text{ N} \cdot \text{m} \\
 \text{F4.4. } \zeta + M_O &= (3 \text{ kN})(1,2 \text{ m} + (0,9 \text{ m}) \cos 45^\circ - 0,3 \text{ m}) \\
 &= 4,61 \text{ kN} \cdot \text{m} \\
 \text{F4.5. } \zeta + M_O &= 50 \sin 60^\circ(0,1 + 0,2 \cos 45^\circ + 0,1) \\
 &\quad - 50 \cos 60^\circ(0,2 \sin 45^\circ) \\
 &= 11,2 \text{ N} \cdot \text{m} \\
 \text{F4.6. } \zeta + M_O &= 500 \sin 45^\circ(3 + 3 \cos 45^\circ) \\
 &\quad - 500 \cos 45^\circ(3 \sin 45^\circ) \\
 &= 1,06 \text{ kN} \cdot \text{m} \\
 \text{F4.7. } \zeta + (M_R)_O &= \Sigma Fd; \\
 (M_R)_O &= -(600 \text{ N})(1 \text{ m}) \\
 &\quad + (500 \text{ N})[3 \text{ m} + (2,5 \text{ m}) \cos 45^\circ] \\
 &\quad - (300 \text{ N})[(2,5 \text{ m}) \sin 45^\circ] \\
 &= 1254 \text{ N} \cdot \text{m} = 1,25 \text{ kN} \cdot \text{m}
 \end{aligned}$$

$$\begin{aligned}
 \text{F4.8. } \zeta + (M_R)_O &= \Sigma Fd; \\
 (M_R)_O &= \left[\left(\frac{3}{5} \right) 500 \text{ N} \right] (0,425 \text{ m}) \\
 &\quad - \left[\left(\frac{4}{5} \right) 500 \text{ N} \right] (0,25 \text{ m}) \\
 &\quad - [(600 \text{ N}) \cos 60^\circ](0,25 \text{ m}) \\
 &\quad - [(600 \text{ N}) \sin 60^\circ](0,425 \text{ m}) \\
 &= -268 \text{ N} \cdot \text{m} = 268 \text{ N} \cdot \text{m} \curvearrowright
 \end{aligned}$$

$$\begin{aligned}
 \text{F4.9. } \zeta + (M_R)_O &= \Sigma Fd; \\
 (M_R)_O &= (1500 \cos 30^\circ \text{ N})(2 \text{ m} + 2 \sin 30^\circ \text{ m}) \\
 &\quad - (1500 \sin 30^\circ \text{ N})(2 \cos 30^\circ \text{ m}) \\
 &\quad + (1000 \text{ N})(2 \cos 30^\circ \text{ m}) \\
 &= 4,33 \text{ kN} \cdot \text{m}
 \end{aligned}$$

$$\begin{aligned}
 \text{F4.10. } \mathbf{F} &= F\mathbf{u}_{AB} = 500 \text{ N} \left(\frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j} \right) = \{400\mathbf{i} - 300\mathbf{j}\} \text{ N} \\
 \mathbf{M}_O &= \mathbf{r}_{OA} \times \mathbf{F} = \{3\mathbf{i}\} \text{ m} \times \{400\mathbf{i} - 300\mathbf{j}\} \text{ N} \\
 &= \{-1200\mathbf{k}\} \text{ N} \cdot \text{m} \\
 &\text{ou} \\
 \mathbf{M}_O &= \mathbf{r}_{OA} \times \mathbf{F} = \{4\mathbf{i}\} \text{ m} \times \{400\mathbf{i} - 300\mathbf{j}\} \text{ N} \\
 &= \{-1200\mathbf{k}\} \text{ N} \cdot \text{m}
 \end{aligned}$$

$$\begin{aligned}
 \text{F4.11. } \mathbf{F} &= F\mathbf{u}_{BC} \\
 &= 600 \text{ N} \left[\frac{\{1,2\mathbf{i} - 1,2\mathbf{j} - 0,6\mathbf{k}\} \text{ m}}{\sqrt{(1,2)^2 + (-1,2)^2 + (-0,6)^2}} \right] \\
 &= \{400\mathbf{i} - 400\mathbf{j} - 200\mathbf{k}\} \text{ N} \\
 \mathbf{M}_O &= \mathbf{r}_C \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1,5 & 0 & 0 \\ 400 & -400 & -200 \end{vmatrix} \\
 &= \{300\mathbf{j} - 600\mathbf{k}\} \text{ N} \cdot \text{m} \\
 &\text{ou} \\
 \mathbf{M}_O &= \mathbf{r}_B \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0,3 & 1,2 & 0,6 \\ 400 & -400 & -200 \end{vmatrix} \\
 &= \{300\mathbf{j} - 600\mathbf{k}\} \text{ N} \cdot \text{m}
 \end{aligned}$$

$$\begin{aligned}
 \text{F4.12. } \mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\
 &= \{(100 - 200)\mathbf{i} + (-120 + 250)\mathbf{j} \\
 &\quad + (75 + 100)\mathbf{k}\} \text{ N} \\
 &= \{-100\mathbf{i} + 130\mathbf{j} + 175\mathbf{k}\} \text{ N} \\
 (M_R)_O &= \mathbf{r}_A \times \mathbf{F}_R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0,8 & 1 & 0,6 \\ -100 & 130 & 175 \end{vmatrix} \\
 &= \{97\mathbf{i} - 200\mathbf{j} + 204\mathbf{k}\} \text{ N} \cdot \text{m}
 \end{aligned}$$

$$\begin{aligned}
 \text{F4.13. } M_x &= \mathbf{i} \cdot (\mathbf{r}_{OB} \times \mathbf{F}) = \begin{vmatrix} 1 & 0 & 0 \\ 0,3 & 0,4 & -0,2 \\ 300 & -200 & 150 \end{vmatrix} \\
 &= 20 \text{ N} \cdot \text{m}
 \end{aligned}$$

$$\mathbf{F4.14.} \quad \mathbf{u}_{OA} = \frac{\mathbf{r}_A}{r_A} = \frac{(0,3\mathbf{i} + 0,4\mathbf{j})}{\sqrt{(0,3\text{ m})^2 + (0,4\text{ m})^2}}$$

$$M_{OA} = \mathbf{u}_{OA} \cdot (\mathbf{r}_{AB} \times \mathbf{F}) = \begin{vmatrix} 0,6 & 0,8 & 0 \\ 0 & 0 & -0,2 \\ 300 & -200 & 150 \end{vmatrix} \\ = -72 \text{ N} \cdot \text{m}$$

$$\mathbf{F4.15.} \quad \mathbf{F} = (200 \text{ N}) \cos 120^\circ \mathbf{i} \\ + (200 \text{ N}) \cos 60^\circ \mathbf{j} + (200 \text{ N}) \cos 45^\circ \mathbf{k} \\ = \{-100\mathbf{i} + 100\mathbf{j} + 141,42\mathbf{k}\} \text{ N}$$

$$\mathbf{M}_O = \mathbf{i} \cdot (\mathbf{r}_A \times \mathbf{F}) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0,3 & 0,25 \\ -100 & 100 & 141,42 \end{vmatrix} \\ = 17,4 \text{ N} \cdot \text{m}$$

$$\mathbf{F4.16.} \quad \mathbf{M}_p = \mathbf{j} \cdot (\mathbf{r}_A \times \mathbf{F}) = \begin{vmatrix} 0 & 1 & 0 \\ -3 & -4 & 2 \\ 30 & -20 & 50 \end{vmatrix} \\ = 210 \text{ N} \cdot \text{m}$$

F4.17.

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{\{-0,4\mathbf{i} + 0,3\mathbf{j}\} \text{ m}}{\sqrt{(-0,4\text{ m})^2 + (0,3\text{ m})^2}} = -0,8\mathbf{i} + 0,6\mathbf{j} \\ M_{AB} = \mathbf{u}_{AB} \cdot (\mathbf{r}_{AC} \times \mathbf{F}) \\ = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0,8 & 0,6 & 0 \\ 0 & 0 & 0,2 \\ 50 & -40 & 20 \end{vmatrix} = -0,4 \text{ N} \cdot \text{m} \\ \mathbf{M}_{AB} = M_{AB} \mathbf{u}_{AB} = \{0,32\mathbf{i} - 0,24\mathbf{j}\} \text{ N} \cdot \text{m}$$

$$\mathbf{F4.18.} \quad F_x = \left[\left(\frac{4}{5} \right) 500 \text{ N} \right] \left(\frac{3}{5} \right) = 240 \text{ N} \\ F_y = \left[\left(\frac{4}{5} \right) 500 \text{ N} \right] \left(\frac{4}{5} \right) = 320 \text{ N} \\ F_z = (500 \text{ N}) \left(\frac{3}{5} \right) = 300 \text{ N} \\ M_x = 300 \text{ N}(2 \text{ m}) - 320 \text{ N}(3 \text{ m}) \\ = -360 \text{ N} \cdot \text{m} \\ M_y = 300 \text{ N}(2 \text{ m}) - 240 \text{ N}(3 \text{ m}) \\ = -120 \text{ N} \cdot \text{m} \\ M_z = 240 \text{ N}(2 \text{ m}) - 320 \text{ N}(2 \text{ m}) \\ = -160 \text{ N} \cdot \text{m}$$

$$\mathbf{F4.19.} \quad \zeta + M_{CR} = \Sigma M_A = 400(3) - 400(5) + 300(5) \\ + 200(0,2) = 740 \text{ N} \cdot \text{m}$$

Também,

$$\zeta + M_{CR} = 300(5) - 400(2) + 200(0,2) \\ = 740 \text{ N} \cdot \text{m}$$

$$\mathbf{F4.20.} \quad \zeta + M_{CR} = 300(0,4) + 200(0,4) + 150(0,4) \\ = 260 \text{ N} \cdot \text{m}$$

$$\mathbf{F4.21.} \quad \zeta + M_{BR} = \Sigma M_B \\ -1,5 \text{ kN} \cdot \text{m} = (2 \text{ kN})(0,3 \text{ m}) - F(0,9 \text{ m}) \\ F = 2,33 \text{ kN}$$

$$\mathbf{F4.22.} \quad \zeta + M_C = 10 \left(\frac{3}{5} \right) (2) - 10 \left(\frac{4}{5} \right) (4) = -20 \text{ kN} \cdot \text{m} \\ = 20 \text{ kN} \cdot \text{m} \curvearrowright$$

$$\mathbf{F4.23.} \quad \mathbf{u}_1 = \frac{\mathbf{r}_1}{r_1} = \frac{[-0,2\mathbf{i} + 0,2\mathbf{j} + 0,35\mathbf{k}] \text{ m}}{\sqrt{(-0,2\text{ m})^2 + (0,2\text{ m})^2 + (0,35\text{ m})^2}} \\ = -\frac{2}{4,5}\mathbf{i} + \frac{2}{4,5}\mathbf{j} + \frac{3,5}{4,5}\mathbf{k} \\ \mathbf{u}_2 = -\mathbf{k}$$

$$\mathbf{u}_3 = \frac{1,5}{2,5}\mathbf{i} - \frac{2}{2,5}\mathbf{j}$$

$$(\mathbf{M}_C)_1 = (\mathbf{M}_C)_1 \mathbf{u}_1$$

$$= (450 \text{ N} \cdot \text{m}) \left(-\frac{2}{4,5}\mathbf{i} + \frac{2}{4,5}\mathbf{j} + \frac{3,5}{4,5}\mathbf{k} \right) \\ = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\} \text{ N} \cdot \text{m}$$

$$(\mathbf{M}_C)_2 = (\mathbf{M}_C)_2 \mathbf{u}_2 = (250 \text{ N} \cdot \text{m})(-\mathbf{k}) \\ = \{-250\mathbf{k}\} \text{ N} \cdot \text{m}$$

$$(\mathbf{M}_C)_3 = (\mathbf{M}_C)_3 \mathbf{u}_3 = (300 \text{ N} \cdot \text{m}) \left(\frac{1,5}{2,5}\mathbf{i} - \frac{2}{2,5}\mathbf{j} \right) \\ = \{180\mathbf{i} - 240\mathbf{j}\} \text{ N} \cdot \text{m}$$

$$(\mathbf{M}_C)_R = \Sigma M_C;$$

$$(\mathbf{M}_C)_R = \{-20\mathbf{i} - 40\mathbf{j} + 100\mathbf{k}\} \text{ N} \cdot \text{m}$$

$$\mathbf{F4.24.} \quad \mathbf{F}_B = \left(\frac{4}{5} \right) (450 \text{ N}) \mathbf{j} - \left(\frac{3}{5} \right) (450 \text{ N}) \mathbf{k} \\ = \{360\mathbf{j} - 270\mathbf{k}\} \text{ N}$$

$$\mathbf{M}_C = \mathbf{r}_{AB} \times \mathbf{F}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0,4 & 0 & 0 \\ 0 & 360 & -270 \end{vmatrix} \\ = \{180\mathbf{j} + 144\mathbf{k}\} \text{ N} \cdot \text{m}$$

também,

$$\mathbf{M}_C = (\mathbf{r}_A \times \mathbf{F}_A) + (\mathbf{r}_B \times \mathbf{F}_B) \\ = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0,3 \\ 0 & -360 & 270 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0,4 & 0 & 0,3 \\ 0 & 360 & -270 \end{vmatrix} \\ = \{180\mathbf{j} + 144\mathbf{k}\} \text{ N} \cdot \text{m}$$

$$\mathbf{F4.25.} \quad \pm F_{Rx} = \Sigma F_x; F_{Rx} = 1000 - \frac{3}{5}(500) = 700 \text{ N}$$

$$+ \downarrow F_{Ry} = \Sigma F_y; F_{Ry} = 750 - \frac{4}{5}(500) = 350 \text{ N}$$

$$F_R = \sqrt{700^2 + 350^2} = 782,6 \text{ N}$$

$$\theta = \text{tg}^{-1} \left(\frac{350}{700} \right) = 26,6^\circ \swarrow$$

$$\zeta + M_{AR} = \Sigma M_A;$$

$$M_{AR} = \frac{3}{5}(500)(1,2) - \frac{4}{5}(500)(1,8) + 750(0,9)$$

$$M_{AR} = 315 \text{ N} \cdot \text{m}$$

$$\text{F4.26. } \pm F_{Rx} = \Sigma F_x; F_{Rx} = \frac{4}{5}(50) = 40 \text{ N}$$

$$+\downarrow F_{Ry} = \Sigma F_y; F_{Ry} = 40 + 30 + \frac{3}{5}(50) = 100 \text{ N}$$

$$F_R = \sqrt{(40)^2 + (100)^2} = 108 \text{ N}$$

$$\theta = \text{tg}^{-1}\left(\frac{100}{40}\right) = 68,2^\circ \swarrow$$

$$\zeta + M_{AR} = \Sigma M_A;$$

$$M_{AR} = 30(3) + \frac{3}{5}(50)(6) + 200 = 470 \text{ N} \cdot \text{m}$$

$$\text{F4.27. } \pm (F_R)_x = \Sigma F_x;$$

$$(F_R)_x = 900 \sin 30^\circ = 450 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y;$$

$$(F_R)_y = -900 \cos 30^\circ - 300 = -1079,42 \text{ N} = 1079,42 \text{ N} \downarrow$$

$$F_R = \sqrt{450^2 + 1079,42^2} = 1169,47 \text{ N} = 1,17 \text{ kN}$$

$$\theta = \text{tg}^{-1}\left(\frac{1079,42}{450}\right) = 67,4^\circ \swarrow$$

$$\zeta + (M_R)_A = \Sigma M_A;$$

$$(M_R)_A = 300 - 900 \cos 30^\circ(0,75) - 300(2,25) = -959,57 \text{ N} \cdot \text{m} = 960 \text{ N} \cdot \text{m} \curvearrowright$$

F4.28.

$$\pm (F_R)_x = \Sigma F_x;$$

$$(F_R)_x = 750\left(\frac{3}{5}\right) + 250 - 500\left(\frac{4}{5}\right) = 300 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y;$$

$$(F_R)_y = -750\left(\frac{4}{5}\right) - 500\left(\frac{3}{5}\right) = -900 \text{ N} \downarrow$$

$$F_R = \sqrt{300^2 + 900^2} = 948,7 \text{ N}$$

$$\theta = \text{tg}^{-1}\left(\frac{900}{300}\right) = 71,6^\circ \swarrow$$

$$\zeta + (M_R)_A = \Sigma M_A;$$

$$(M_R)_A = 500\left(\frac{4}{5}\right)(0,3) - 500\left(\frac{3}{5}\right)(1,8) - 750\left(\frac{4}{5}\right)(0,9) = -960 = 960 \text{ N} \cdot \text{m} \curvearrowright$$

$$\text{F4.29. } F_R = \Sigma F;$$

$$F_R = F_1 + F_2$$

$$= (-300\mathbf{i} + 150\mathbf{j} + 200\mathbf{k}) + (-450\mathbf{k})$$

$$= \{-300\mathbf{i} + 150\mathbf{j} - 200\mathbf{k}\} \text{ N}$$

$$\mathbf{r}_{OA} = (2 - 0)\mathbf{j} = \{2\mathbf{j}\} \text{ m}$$

$$\mathbf{r}_{OB} = (-1,5 - 0)\mathbf{i} + (2 - 0)\mathbf{j} + (1 - 0)\mathbf{k}$$

$$= \{-1,5\mathbf{i} + 2\mathbf{j} + 1\mathbf{k}\} \text{ m}$$

$$(\mathbf{M}_R)_O = \Sigma \mathbf{M};$$

$$(\mathbf{M}_R)_O = \mathbf{r}_{OB} \times \mathbf{F}_1 + \mathbf{r}_{OA} \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1,5 & 2 & 1 \\ -300 & 150 & 200 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 0 \\ 0 & 0 & -450 \end{vmatrix} = \{-650\mathbf{i} + 375\mathbf{k}\} \text{ N} \cdot \text{m}$$

$$\text{F4.30. } F_1 = \{-100\mathbf{j}\} \text{ N}$$

$$F_2 = (200 \text{ N}) \left[\frac{\{-0,4\mathbf{i} - 0,3\mathbf{k}\} \text{ m}}{\sqrt{(-0,4 \text{ m})^2 + (-0,3 \text{ m})^2}} \right] = \{-160\mathbf{i} - 120\mathbf{k}\} \text{ N}$$

$$\mathbf{M}_C = \{-75\mathbf{i}\} \text{ N} \cdot \text{m}$$

$$\mathbf{F}_R = \{-160\mathbf{i} - 100\mathbf{j} - 120\mathbf{k}\} \text{ N}$$

$$(\mathbf{M}_R)_O = (0,3\mathbf{k}) \times (-100\mathbf{j})$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0,5 & 0,3 \\ -160 & 0 & -120 \end{vmatrix} + (-75\mathbf{i}) = \{-105\mathbf{i} - 48\mathbf{j} + 80\mathbf{k}\} \text{ N} \cdot \text{m}$$

$$\text{F4.31. } +\downarrow F_R = \Sigma F_y; F_R = 2,5 + 1,25 + 2,5 = 6,25 \text{ kN}$$

$$\zeta + F_R x = \Sigma M_O;$$

$$6,25(x) = 2,5(1) + 1,25(2) + 2,5(3)$$

$$x = 2 \text{ m}$$

$$\text{F4.32. } \pm (F_R)_x = \Sigma F_x;$$

$$(F_R)_x = 0,5\left(\frac{3}{5}\right) + 0,25 \sin 30^\circ = 0,425 \text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y;$$

$$(F_R)_y = 1 + 0,25 \cos 30^\circ - 0,5\left(\frac{4}{5}\right) = 0,8165 \text{ kN} \uparrow$$

$$F_R = \sqrt{0,425^2 + 0,8165^2} = 0,917 \text{ N}$$

$$\theta = \text{tg}^{-1}\left(\frac{0,8165}{0,425}\right) = 62,5^\circ \nearrow$$

$$\zeta + (M_R)_A = \Sigma M_A;$$

$$0,8165(d) = 1(1) - 0,5\left(\frac{4}{5}\right)(2) + 0,25 \cos 30^\circ(3)$$

$$d = 1,04 \text{ m}$$

$$\begin{aligned}
 \text{F4.33. } \pm (F_R)_x &= \Sigma F_x; \\
 (F_R)_x &= 15\left(\frac{4}{5}\right) = 12 \text{ kN} \rightarrow \\
 +\uparrow (F_R)_y &= \Sigma F_y; \\
 (F_R)_y &= -20 + 15\left(\frac{3}{5}\right) = -11 \text{ kN} = 11 \text{ kN} \downarrow \\
 F_R &= \sqrt{12^2 + 11^2} = 16,3 \text{ kN} \\
 \theta &= \text{tg}^{-1}\left(\frac{11}{12}\right) = 42,5^\circ \swarrow \\
 \zeta + (M_R)_A &= \Sigma M_A; \\
 -11(d) &= -20(2) - 15\left(\frac{4}{5}\right)(2) + 15\left(\frac{3}{5}\right)(6) \\
 d &= 0,909 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{F4.34. } \pm (F_R)_x &= \Sigma F_x; \\
 (F_R)_x &= \left(\frac{3}{5}\right)5 \text{ kN} - 8 \text{ kN} \\
 &= -5 \text{ kN} = 5 \text{ kN} \rightarrow \\
 +\uparrow (F_R)_y &= \Sigma F_y; \\
 (F_R)_y &= -6 \text{ kN} - \left(\frac{4}{5}\right)5 \text{ kN} \\
 &= -10 \text{ kN} = 10 \text{ kN} \downarrow \\
 F_R &= \sqrt{5^2 + 10^2} = 11,2 \text{ kN} \\
 \theta &= \text{tg}^{-1}\left(\frac{10 \text{ kN}}{5 \text{ kN}}\right) = 63,4^\circ \searrow \\
 \zeta + (M_R)_A &= \Sigma M_A; \\
 5 \text{ kN}(d) &= 8 \text{ kN}(3 \text{ m}) - 6 \text{ kN}(0,5 \text{ m}) \\
 &\quad - \left[\left(\frac{4}{5}\right)5 \text{ kN}\right](2 \text{ m}) \\
 &\quad - \left[\left(\frac{3}{5}\right)5 \text{ kN}\right](4 \text{ m}) \\
 d &= 0,2 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{F4.35. } +\downarrow F_R &= \Sigma F_z; \quad F_R = 400 + 500 - 100 \\
 &= 800 \text{ N} \\
 M_{Rx} &= \Sigma M_x; \quad -800y = -400(4) - 500(4) \\
 y &= 4,50 \text{ m} \\
 M_{Ry} &= \Sigma M_y; \quad 800x = 500(4) - 100(3) \\
 x &= 2,125 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{F4.36. } +\downarrow F_R &= \Sigma F_z; \\
 F_R &= 200 + 200 + 100 + 100 \\
 &= 600 \text{ N} \\
 M_{Rx} &= \Sigma M_x; \\
 -600y &= 200(1) + 200(1) + 100(3) - 100(3) \\
 y &= -0,667 \text{ m} \\
 M_{Ry} &= \Sigma M_y; \\
 600x &= 100(3) + 100(3) + 200(2) - 200(3) \\
 x &= 0,667 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{F4.37. } +\uparrow F_R &= \Sigma F_y; \\
 -F_R &= -6(1,5) - 9(3) - 3(1,5) \\
 F_R &= 40,5 \text{ kN} \downarrow \\
 \zeta + (M_R)_A &= \Sigma M_A; \\
 &= -40,5(d) = 6(1,5)(0,75) \\
 &\quad - 9(3)(1,5) - 3(1,5)(3,75) \\
 d &= 1,25 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{F4.38. } F_R &= \frac{1}{2}(1,8)(3) + 2,4(3) = 9,9 \text{ kN} \\
 \zeta + M_{AR} &= \Sigma M_A; \\
 9,9d &= \left[\frac{1}{2}(1,8)(3)\right](1,2) + [2,4(3)](3) \\
 d &= 2,51 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{F4.39. } +\uparrow F_R &= \Sigma F_y; \\
 -F_R &= -\frac{1}{2}(6)(3) - \frac{1}{2}(6)(6) \\
 F_R &= 27 \text{ kN} \downarrow \\
 \zeta + (M_R)_A &= \Sigma M_A; \\
 -27(d) &= \frac{1}{2}(6)(3)(1) - \frac{1}{2}(6)(6)(2) \\
 d &= 1 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{F4.40. } +\downarrow F_R &= \Sigma F_y; \\
 F_R &= \frac{1}{2}(1)(2) + 3(2) + 2,5 \\
 &= 9,5 \text{ kN} \\
 \zeta + M_{AR} &= \Sigma M_A; \\
 9,5d &= \left[\frac{1}{2}(1)(2)\right]\left(2\right)\left(\frac{2}{3}\right) + [3(2)](1) + 2,5(3) \\
 d &= 1,56 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{F4.41. } +\uparrow F_R &= \Sigma F_y; \\
 -F_R &= -\frac{1}{2}(3)(4,5) - 3(6) \\
 F_R &= 24,75 \text{ kN} \downarrow \\
 \zeta + (M_R)_A &= \Sigma M_A; \\
 -24,75(d) &= -\frac{1}{2}(3)(4,5)(1,5) - 3(6)(3) \\
 d &= 2,59 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{F4.42. } F_R &= \int_0^4 w(x) dx = \int_0^4 2,5x^3 dx = 160 \text{ N} \\
 \zeta + M_{AR} &= \Sigma M_A; \\
 x &= \frac{\int xw(x) dx}{\int w(x) dx} = \frac{\int_0^4 2,5x^4 dx}{160} = 3,20 \text{ m}
 \end{aligned}$$

Capítulo 5

F5.1. $\pm \Sigma F_x = 0; -A_x + 2500\left(\frac{3}{5}\right) = 0$

$$A_x = 1500 \text{ N}$$

$\zeta + \Sigma M_A = 0; B_y(3) - 2500\left(\frac{4}{5}\right)(1,5) - 900 = 0$

$$B_y = 1300 \text{ N}$$

$+ \uparrow \Sigma F_y = 0; A_y + 1300 - 2500\left(\frac{4}{5}\right) = 0$

$$A_y = 700 \text{ N}$$

F5.2. $\zeta + \Sigma M_A = 0;$

$$F_{CD} \text{ sen } 45^\circ(1,5 \text{ m}) - 4 \text{ kN}(3 \text{ m}) = 0$$

$$F_{CD} = 11,31 \text{ kN} = 11,3 \text{ kN}$$

$\pm \Sigma F_x = 0; A_x + (11,31 \text{ kN}) \cos 45^\circ = 0$

$$A_x = -8 \text{ kN} = 8 \text{ kN} \leftarrow$$

$+ \uparrow \Sigma F_y = 0;$

$$A_y + (11,31 \text{ kN}) \text{ sen } 45^\circ - 4 \text{ kN} = 0$$

$$A_y = -4 \text{ kN} = 4 \text{ kN} \downarrow$$

F5.3. $\zeta + \Sigma M_A = 0;$

$$N_B[6 \text{ m} + (6 \text{ m}) \cos 45^\circ]$$

$$- 10 \text{ kN}[2 \text{ m} + (6 \text{ m}) \cos 45^\circ]$$

$$- 5 \text{ kN}(4 \text{ m}) = 0$$

$$N_B = 8,047 \text{ kN} = 8,05 \text{ kN}$$

$\pm \Sigma F_x = 0;$

$$(5 \text{ kN}) \cos 45^\circ - A_x = 0$$

$$A_x = 3,54 \text{ kN}$$

$+ \uparrow \Sigma F_y = 0;$

$$A_y + 8,047 \text{ kN} - (5 \text{ kN}) \text{ sen } 45^\circ - 10 \text{ kN} = 0$$

$$A_y = 5,49 \text{ kN}$$

F5.4. $\pm \Sigma F_x = 0; -A_x + 400 \cos 30^\circ = 0$

$$A_x = 346 \text{ N}$$

$+ \uparrow \Sigma F_y = 0;$

$$A_y = -200 - 200 - 200 - 400 \text{ sen } 30^\circ = 0$$

$$A_y = 800 \text{ N}$$

$\zeta + \Sigma M_A = 0;$

$$M_A - 200(2,5) - 200(3,5) - 200(4,5)$$

$$- 400 \text{ sen } 30^\circ(4,5) - 400 \cos 30^\circ(3 \text{ sen } 60^\circ) = 0$$

$$M_A = 3,90 \text{ kN} \cdot \text{m}$$

F5.5. $\zeta + \Sigma M_A = 0;$

$$N_C(0,7 \text{ m}) - [25(9,81) \text{ N}](0,5 \text{ m}) \cos 30^\circ = 0$$

$$N_C = 151,71 \text{ N} = 152 \text{ N}$$

$\pm \Sigma F_x = 0;$

$$T_{AB} \cos 15^\circ - (151,71 \text{ N}) \cos 60^\circ = 0$$

$$T_{AB} = 78,53 \text{ N} = 78,5 \text{ N}$$

$+ \uparrow \Sigma F_y = 0;$

$$F_A + (78,53 \text{ N}) \text{ sen } 15^\circ$$

$$+ (151,71 \text{ N}) \text{ sen } 60^\circ - 25(9,81) \text{ N} = 0$$

$$F_A = 93,5 \text{ N}$$

F5.6. $\pm \Sigma F_x = 0;$

$$N_C \text{ sen } 30^\circ - (250 \text{ N}) \text{ sen } 60^\circ = 0$$

$$N_C = 433,0 \text{ N} = 433 \text{ N}$$

$\zeta + \Sigma M_B = 0;$

$$- N_A \text{ sen } 30^\circ(0,15 \text{ m}) - 433,0 \text{ N}(0,2 \text{ m})$$

$$+ [(250 \text{ N}) \cos 30^\circ](0,6 \text{ m}) = 0$$

$$N_A = 577,4 \text{ N} = 577 \text{ N}$$

$+ \uparrow \Sigma F_y = 0;$

$$N_B - 577,4 \text{ N} + (433,0 \text{ N}) \cos 30^\circ$$

$$- (250 \text{ N}) \cos 60^\circ = 0$$

$$N_B = 327 \text{ N}$$

F5.7. $\Sigma F_z = 0;$

$$T_A + T_B + T_C - 1 - 2,5 = 0$$

$\Sigma M_x = 0;$

$$T_A(0,9) + T_C(0,9) - 2,5(0,45) - 1(0,9) = 0$$

$\Sigma M_y = 0;$

$$- T_B(0,1,2) - T_C(0,1,2) + 2,5(0,6) + 1(0,6) = 0$$

$$T_A = 1,75 \text{ kN}, T_B = 1,25 \text{ kN}, T_C = 0,5 \text{ kN}$$

F5.8. $\Sigma M_y = 0;$

$$600 \text{ N}(0,2 \text{ m}) + 900 \text{ N}(0,6 \text{ m}) - F_A(1 \text{ m}) = 0$$

$$F_A = 660 \text{ N}$$

$\Sigma M_x = 0;$

$$D_z(0,8 \text{ m}) - 600 \text{ N}(0,5 \text{ m}) - 900 \text{ N}(0,1 \text{ m}) = 0$$

$$D_z = 487,5 \text{ N}$$

$\Sigma F_x = 0; D_x = 0$

$\Sigma F_y = 0; D_y = 0$

$\Sigma F_z = 0;$

$$T_{BC} + 660 \text{ N} + 487,5 \text{ N} - 900 \text{ N} - 600 \text{ N} = 0$$

$$T_{BC} = 352,5 \text{ N}$$

F5.9. $\Sigma F_y = 0; 400 \text{ N} + C_y - 0;$

$$C_y = -400 \text{ N}$$

$\Sigma M_y = 0; -C_x(0,4 \text{ m}) - 600 \text{ N}(0,6 \text{ m}) = 0$

$$C_x = -900 \text{ N}$$

$$\Sigma M_x = 0; B_z(0,6 \text{ m}) + 600 \text{ N}(1,2 \text{ m})$$

$$+ (-400 \text{ N})(0,4 \text{ m}) = 0$$

$$B_z = -933,3 \text{ N}$$

$$\Sigma M_z = 0;$$

$$-B_x(0,6 \text{ m}) + (-900 \text{ N})(1,2 \text{ m})$$

$$+ (-400 \text{ N})(0,6 \text{ m}) = 0$$

$$B_x = 1400 \text{ N}$$

$$\Sigma F_x = 0; 1400 \text{ N} + (-900 \text{ N}) - A_x = 0$$

$$A_x = 500 \text{ N}$$

$$\Sigma F_z = 0; A_z - 933,3 \text{ N} + 600 \text{ N} = 0$$

$$A_z = 333,3 \text{ N}$$

F5.10. $\Sigma F_x = 0; B_x = 0$

$$\Sigma M_z = 0;$$

$$C_y(0,4 \text{ m} + 0,6 \text{ m}) = 0 \quad C_y = 0$$

$$\Sigma F_y = 0; A_y + 0 = 0 \quad A_y = 0$$

$$\Sigma M_x = 0; C_y(0,6 \text{ m} + 0,6 \text{ m}) + B_z(0,6 \text{ m})$$

$$-450 \text{ N}(0,6 \text{ m} + 0,6 \text{ m}) = 0$$

$$1,2C_z + 0,6B_z - 540 = 0$$

$$\Sigma M_y = 0; -C_z(0,6 \text{ m} + 0,4 \text{ m})$$

$$-B_z(0,6 \text{ m}) + 450 \text{ N}(0,6 \text{ m}) = 0$$

$$-C_z - 0,6B_z + 270 = 0$$

$$C_z = 1350 \text{ N} \quad B_z = -1800 \text{ N}$$

$$\Sigma F_z = 0;$$

$$A_z + 1350 \text{ N} + (-1800 \text{ N}) - 450 \text{ N} = 0$$

$$A_z = 900 \text{ N}$$

F5.11. $\Sigma F_y = 0; A_y = 0$

$$\Sigma M_x = 0; -9(3) + F_{CE}(3) = 0$$

$$F_{CE} = 9 \text{ kN}$$

$$\Sigma M_z = 0; F_{CF}(3) - 6(3) = 0$$

$$F_{CF} = 6 \text{ kN}$$

$$\Sigma M_y = 0; 9(4) - A_z(4) - 6(1,5) = 0$$

$$A_z = 6,75 \text{ kN}$$

$$\Sigma F_x = 0; A_x + 6 - 6 = 0 \quad A_x = 0$$

$$\Sigma F_z = 0; F_{DB} + 9 - 9 + 6,75 = 0$$

$$F_{DB} = -6,75 \text{ kN}$$

F5.12. $\Sigma F_x = 0; A_x = 0$

$$\Sigma F_y = 0; A_y = 0$$

$$\Sigma F_z = 0; A_z + F_{BC} - 400 = 0$$

$$\Sigma M_x = 0; (M_A)_x + 1,8F_{BC} - 400(1,8) = 0$$

$$\Sigma M_y = 0; 9F_{BC} - 400(0,45) = 0 \quad F_{BC} = 200 \text{ N}$$

$$\Sigma M_z = 0; (M_A)_z = 0$$

$$A_z = 200 \text{ N} \quad (M_A)_x = 360 \text{ N} \cdot \text{m}$$

Capítulo 6

F6.1. *Nó A.*

$$+\uparrow \Sigma F_y = 0; 1 \text{ kN} - F_{AD} \sin 45^\circ = 0$$

$$F_{AD} = 1,414 \text{ kN}(C)$$

$$\pm \Sigma F_x = 0; F_{AB} - (1,414 \text{ kN}) \cos 45^\circ = 0$$

$$F_{AB} = 1 \text{ kN}(T)$$

Nó B.

$$\pm \Sigma F_x = 0; F_{BC} - 1 \text{ kN} = 0$$

$$F_{BC} = 1 \text{ kN}(T)$$

$$+\uparrow \Sigma F_y = 0; F_{BD} = 0$$

Nó D.

$$\pm \Sigma F_x = 0;$$

$$F_{CD} \cos 45^\circ + (1,414 \text{ kN}) \cos 45^\circ - 2 \text{ kN} = 0$$

$$F_{CD} = 1,414 \text{ kN}(T)$$

F6.2. *Nó D:*

$$+\uparrow \Sigma F_y = 0; \frac{3}{5}F_{CD} - 1,5 \text{ kN} = 0$$

$$F_{CD} = 2,5 \text{ kN}(T)$$

$$\pm \Sigma F_x = 0; -F_{AD} + \frac{4}{5}(2,5) = 0$$

$$F_{AD} = 2 \text{ kN}(C)$$

$$F_{BC} = 2,5 \text{ kN}(T), F_{AC} = F_{AB} = 0$$

F6.3. $A_x = 0, A_y = C_y = 2 \text{ kN}$

Nó A:

$$+\uparrow \Sigma F_y = 0; -\frac{3}{5}F_{AE} + 2 = 0$$

$$F_{AE} = 3,333 \text{ kN}(C)$$

Nó C:

$$\uparrow F_y = 0; -F_{DC} + 2 = 0;$$

$$F_{DC} = 2 \text{ kN}(C)$$

F6.4. *Nó C.*

$$+\uparrow \Sigma F_y = 0; 2F \cos 30^\circ - P = 0$$

$$F_{AC} = F_{BC} = F = \frac{P}{2 \cos 30^\circ} = 0,5774P(C)$$

Nó B.

$$\pm \Sigma F_x = 0; 0,5774P \cos 60^\circ - F_{AB} = 0$$

$$F_{AB} = 0,2887P(T)$$

$$F_{AB} = 0,2887P = 2 \text{ kN}$$

$$P = 6,928 \text{ kN}$$

$$F_{AC} = F_{BC} = 0,5774P = 1,5 \text{ kN}$$

$$P = 2,598 \text{ kN}$$

O menor valor de P é escolhido

$$P = 2,598 \text{ kN} = 2,60 \text{ kN}$$

F6.5. $F_{CB} = 0$

$F_{CD} = 0$

$F_{AE} = 0$

$F_{DE} = 0$

F6.6. *Nô C.*

$$+\uparrow \Sigma F_y = 0; \quad 1,299 \text{ kN} - F_{CD} \text{ sen } 30^\circ = 0$$

$F_{CD} = 2,598 \text{ kN (C)}$

$$\pm \Sigma F_x = 0; \quad (2,598 \text{ kN}) \cos 30^\circ - F_{BC} = 0$$

$F_{BC} = 2,25 \text{ kN (T)}$

Nô D.

$$+ \nearrow \Sigma F_y = 0; \quad F_{BD} \cos 30^\circ = 0 \quad F_{BD} = 0$$

$$+ \searrow \Sigma F_x = 0; \quad F_{DE} - 2,598 \text{ kN} = 0$$

$F_{DE} = 2,598 \text{ kN (C)}$

Nô B.

$$+\uparrow \Sigma F_y = 0; \quad F_{BE} \text{ sen } \phi = 0 \quad F_{BE} = 0$$

$$\pm \Sigma F_x = 0; \quad 2,25 \text{ kN} - F_{AB} = 0$$

$F_{AB} = 2,25 \text{ kN (T)}$

Nô A

$$+\uparrow \Sigma F_y = 0; \quad 1,701 \text{ kN} - F_{AE} = 0$$

$F_{AE} = 1,701 \text{ kN (C)}$

F6.7. $+\uparrow \Sigma F_y = 0; \quad F_{CF} \text{ sen } 45^\circ - 3 - 4 = 0$

$F_{CF} = 9,899 \text{ kN (T)}$

$$\pm \Sigma M_C = 0; \quad F_{FE}(1) - 4(1) = 0$$

$F_{FE} = 4 \text{ kN (T)}$

$$\pm \Sigma M_F = 0; \quad F_{BC}(1) - 3(1) - 4(2) = 0$$

$F_{BC} = 11 \text{ kN (C)}$

F6.8. $+\uparrow \Sigma F_y = 0; \quad F_{KC} + 33,33 \text{ kN} - 40 \text{ kN} = 0$

$F_{KC} = 6,67 \text{ kN (C)}$

$$\pm \Sigma M_K = 0;$$

$$33,33 \text{ kN (8 m)} - 40 \text{ kN (2 m)} - F_{CD}(3 \text{ m}) = 0$$

$F_{CD} = 62,22 \text{ kN (T)}$

$$\pm \Sigma F_x = 0; \quad F_{LK} - 62,22 \text{ kN} = 0$$

$F_{LK} = 62,22 \text{ kN (C)}$

F6.9. $\pm \Sigma M_A = 0; \quad G_y(12 \text{ m}) - 20 \text{ kN (2 m)}$

$$- 30 \text{ kN (4 m)} - 40 \text{ kN (6 m)} = 0$$

$G_y = 33,33 \text{ kN}$

Pela geometria da treliça,

$$\phi = \text{tg}^{-1}(3 \text{ m/2 m}) = 56,31^\circ$$

$$\pm \Sigma M_K = 0;$$

$$33,33 \text{ kN (8 m)} - 40 \text{ kN (2 m)} - F_{CD}(3 \text{ m}) = 0$$

$F_{CD} = 62,22 \text{ kN (T)}$

$$\pm \Sigma M_D = 0; \quad 33,33 \text{ kN (6 m)} - F_{KD}(3 \text{ m}) = 0$$

$F_{KJ} = 66,7 \text{ kN (C)}$

$$+\uparrow \Sigma F_y = 0;$$

$$33,33 \text{ kN} - 40 \text{ kN} + F_{KD} \text{ sen } 56,31^\circ = 0$$

$F_{KD} = 8,01 \text{ kN (T)}$

F6.10. Pela geometria da treliça,

$$\text{tg } \phi = \frac{1 \text{ m}}{(3 \text{ m}) \text{tg } 30^\circ} = 1,732 \quad \phi = 60^\circ$$

$$\pm \Sigma M_C = 0;$$

$$F_{EF} \text{ sen } 30^\circ (2 \text{ m}) + 1,5 \text{ kN (2 m)} = 0$$

$F_{EF} = -3 \text{ kN (C)}$

$$\pm \Sigma M_D = 0;$$

$$1,5 \text{ kN (2 m)} - F_{CF} \text{ sen } 60^\circ (2 \text{ m}) = 0$$

$F_{CF} = 1,732 \text{ kN (T)}$

$$\pm \Sigma M_F = 0;$$

$$1,5 \text{ kN (3 m)} - 1,5 \text{ kN (1 m)} - F_{BC}(3 \text{ m}) \text{tg } 30^\circ = 0$$

$F_{BC} = 1,732 \text{ kN (T)}$

F6.11. Pela geometria da treliça,

$$\theta = \text{tg}^{-1}(1 \text{ m/2 m}) = 26,57^\circ$$

$$\phi = \text{tg}^{-1}(3 \text{ m/2 m}) = 56,31^\circ$$

O local de G pode ser achado usando triângulos semelhantes

$$\frac{1 \text{ m}}{2 \text{ m}} = \frac{2 \text{ m} + x}{4 \text{ m}}$$

$$4 \text{ m} = 2 \text{ m} + x$$

$$x = 2 \text{ m}$$

$$\pm \Sigma M_G = 0;$$

$$26,25 \text{ kN (4 m)} - 15 \text{ kN (2 m)} - F_{CD}(3 \text{ m}) = 0$$

$F_{CD} = 25 \text{ kN (T)}$

$$\pm \Sigma M_D = 0;$$

$$26,25 \text{ kN (2 m)} - F_{GF} \cos 26,57^\circ (2 \text{ m}) = 0$$

$F_{GF} = 29,3 \text{ kN (C)}$

$$\pm \Sigma M_O = 0; \quad 15 \text{ kN (4 m)} - 26,25 \text{ kN (2 m)}$$

$$- F_{GD} \text{ sen } 56,31^\circ (4 \text{ m}) = 0$$

$$F_{GD} = 22,25 \text{ kN (T)}$$

F6.12. $\pm \Sigma M_H = 0;$

$$F_{DC}(4 \text{ m}) + 8 \text{ kN (7 m)} = 0$$

$F_{DC} = 9,5 \text{ kN (C)}$

$$\pm \Sigma M_D = 0;$$

$$6 \text{ kN (7 m)} - 8 \text{ kN (3 m)} - F_{HI}(4 \text{ m}) = 0$$

$F_{HI} = 4,5 \text{ kN (C)}$

$$\pm \Sigma M_C = 0; \quad F_{JI} \cos 45^\circ (4 \text{ m}) + 6 \text{ kN (7 m)}$$

$$- 4,5 \text{ kN (4 m)} - 8 \text{ kN (3 m)}$$

$$F_{HI} = 0$$